AN INTEGRATED INTERVAL ARITHMETIC APPROACH FOR REACTIVE POWER COMPENSATION OF RADIAL DISTRIBUTION FEEDERS

M. Abouelsaad Mohamed Anwar
Department of Electrical Engineering– Shoubra Faculty of Engineering, University of Benha– Egypt
mabouelsaad@yahoo.com moh_an1@yahoo,com

ABSTRACT

The paper presents the application of Interval Mathematics as a new method to, rigorously, address uncertainties associated with capacitor installation in distribution feeders. While several methods exist to determine optimal capacitor sizes and locations in distribution feeders, these methods usually require data which may be uncertain in nature. To account for such uncertainties a heuristic method coupled with interval mathematics is developed with the aim of maximizing the cost saving by placing single node-located capacitors at a selected sequence of nodes. The method determines a set of nodes to be compensated with their associated interval capacitor sizes; followed by a local search loop, at any one node, to determine the final standard capacitor size at this node. While catering for uncertainties, the method also offers utilities with alternatives for selecting the standard capacitor sizes to be used and the associated costs to be saved. This should enable utilities to make informed decisions regarding installing capacitors for reactive power compensation in their distribution systems. A procedure is devised in order to produce sharp bounds of the interval outcomes. Successful implementation of the proposed method is described using a nine buses example distribution feeder.

INTRODUCTION

Energy management through reactive power compensation on distribution systems has, recently, emerged as a topic of current research interest [1-4]. Reactive power flow in a distribution system produces losses and results in increased rating for the system components. Shunt capacitors are usually installed to reduce these power losses, increase the released thermal capacities of the lines and transformers and improve the system voltage profile.

However, the data employed in the reactive power compensation analysis is usually derived from many sources with varying degrees of accuracy. Accounting for such uncertainties is necessary to produce realistic results which utilities can employ to make informed decisions regarding reactive power compensation in their distribution systems.

Uncertainties can be looked upon as a condition in which the possibility of errors exists as a result of having less than total information about the surrounding environment. They are beyond the utility's foreknowledge or control. In a distribution system, the reactive load is always varying and it is not a realistic proposition to determine capacitor sizes and locations based on an average of the reactive loads as even this number is subject to change as the load varies. In addition, many of the reactive power compensation techniques involves the optimization of a cost function which require parameters such as the cost of the capacitors, the cost of energy and the cost of the peak power savings to which only an estimation (single-point) without exact certainty can be obtained [3]. Consequently, the validity of the results generated is questionable.

Interval mathematics provides a powerful tool for the implementation and extension of the "unknown but bounded" concept [5,6]. Using interval analysis, there is no need for many simulation runs as the total variation of the solution considers the simultaneous variations of all inputs in a single run. In this form of mathematics, interval numbers are used instead of single point numbers.

This paper presents an interval method coupled with a heuristic technique for maximizing the cost saving; by placing optimal capacitors at proper locations in interval format. Uncertainties in the parameters are integrated into the analysis, as interval numbers, to allocate, sequentially, the capacitors according to the upper limit of the maximum interval saving outcome. Once locations are identified, the standard capacitor size, at a selected location, is determined through the optimization of the cost saving function. The method offers utilities with alternatives for selecting the standard capacitor sizes to be used and the associated costs to be saved. To overcome the difficulty of conservative bounds, a procedure is devised in order to produce sharp bounds of the interval outcomes and consequently enhances the decision making process. The proposed method is tested on a nine-bus distribution feeder and encouraging results are reported.

THE GOVERNING EQUATIONS

In order to account for uncertainties associated with the capacitors sizing and location problem, the maximum cost saving analysis is followed [4]. The input parameters' uncertainties, in interval format, are integrated into the governing equations as follows:

$$P = \sum_{i=1}^{n} I_i^2 R_i \tag{1}$$

where P is the total active power loss for a distribution system with n branches, I_i and R_i are the current magnitude and resistance, respectively of branch i. The branch current can be obtained from the load flow solution. This current has two components; active (I_a) and reactive (I_r). Thus, the system losses can be written as

$$P = \sum_{i=1}^{n} I_{ai}^2 R_i + \sum_{i=1}^{n} I_{ri}^2 R_i \tag{2}$$

If a capacitor of current I_{ck} is placed at a node k, the system losses are

$$P = \sum_{i=1}^{n} I_{ai}^2 R_i + \sum_{i=1}^{k} (I_{ri} + I_{ck})^2 R_i + \sum_{i=k+1}^{n} I_{ri}^2 R_i \tag{3}$$

Subtracting (3) from (2), the loss reduction ΔP_k is

$$\Delta P_k = -2I_{ck}\sum_{i=1}^{k} I_{ri}R_i - I_{ck}^2\sum_{i=1}^{k}R_i \tag{4}$$

Assuming there is no significant change in the node voltage after setting the capacitor and using the cost function equation, the cost reduction can be defined as

$$\Delta S = K_p\Delta P + K_e\Delta E - K_{ck}Q_{ck} \tag{5}$$

where K_p is the annual cost in \$/KW and K_{ck} is the annual cost in \$/KVAr for the capacitor placed at node k both represented in interval format. K_e is the interval annual cost of KWh losses in \$/KWh with the energy losses, defined over a time period T, using (4) as

$$\Delta E_k = -2L_f TI_{ck}\sum_{i=1}^{k} I_{ri}R_i - TI_{ck}^2\sum_{i=1}^{k}R_i \tag{6}$$

where L_f is the interval load factor. Q_{ck} is the capacitor size at node k and equals

$$Q_{ck} = I_{ck}V_k \tag{7}$$

Substituting for (4), (6-7) in (5), we get

$$\Delta S = -2K_p^{'}I_{ck}\sum_{i=1}^{k} I_{ri}R_i - K_p^{"}I_{ck}^2\sum_{i=1}^{k}R_i \tag{8}$$

where

$$K_p^{'} = K_p + K_e TL_f + \frac{K_{ck}V_k}{2\sum_{i=1}^{k} I_{ri}R_i} \tag{9}$$

$$K_p^{"} = K_p + K_e T$$

The value of I_{ck} that maximizes the cost reduction is obtained by

$$\frac{\partial \Delta S}{\partial I_{ck}} = 0 \tag{10}$$

From (9), we get the interval I_{ck} as

$$I_{ck} = -\frac{K_p^{'}}{K_p^{"}}\frac{\sum_{i=1}^{k} I_{ri}R_i}{\sum_{i=1}^{k} R_i} \tag{11}$$

Substituting from (11) into (8) and (4), we obtain the interval maximum net saving and the corresponding interval loss reduction as follows:

$$\Delta S_{\max} = \frac{(K_p^{'})^2}{K_p^{"}}\frac{(\sum_{i=1}^{k} I_{ri}R_i)^2}{\sum_{i=1}^{k} R_i} \tag{12}$$

$$\Delta P_{\Delta S\max} = (\frac{K_p^{'}}{K_p^{"}})(2-\frac{K_p^{'}}{K_p^{"}})\frac{(\sum_{i=1}^{k} I_{ri}R_i)^2}{\sum_{i=1}^{k} R_i} \tag{13}$$

Using (7) and (11-13), respectively, we can calculate the size of the capacitor used at a certain node k that maximizes the total system cost reduction and we can compute the interval maximum cost reduction as well as the corresponding interval loss reduction.

INTERVAL MATHEMATICS

Interval mathematics provides a useful tool in determining the effects of uncertainty in parameters used in a computation. In this form of mathematics, interval numbers are used instead of ordinary single point numbers. An interval number is defined as an ordered pair of real numbers representing the lower and upper bounds of the parameter range [6]. An interval number can then be formally defined as follows; $[a, b]$, where $a \le b$.

Given two interval numbers, $[a, b]$ and $[c, d]$, the rules for interval addition, subtraction, multiplication, and division are as follows:

$$\begin{aligned} &[a,b]+[c,d]=[a+c,b+d] \\ &[a,b]-[c,d]=[a-d,b-c] \\ &[a,b]*[c,d]=[\min(ac,ad,bc,bd),\max(ac,ad,bc,bd)] \\ &[a,b]/[c,d]=[a,b]*[1/d,1/c],\text{where } 0\notin[c,d] \end{aligned} \tag{14}$$

Implementing interval analysis techniques confronts some obstacles because its algebraic structure is unlike that of common single point arithmetic. Accordingly, interval computations may produce wide bounds. The bounds of the resulting interval computations may depend on the calculation procedure as well as the input parameters. So, an effort has to be made to reduce the width of the resulting interval bounds. Normally, the approach to producing better bounds has been to rearrange the expression to reduce the appearance of the interval parameters [6].

UNCERTAINTIES IN THE CAPACITOR SIZING AND PLACEMENT PROBLEM

Inspection of equations (7-13) reveals that it is likely that values for K_p, K_e, K_{ck} and L_f can not be obtained with absolute certainty. For instance, K_p and K_e , the costs for the peak power and energy losses respectively can be calculated in many ways but it is probably known that there is an upper and lower bound for these costs which can be attributed with more certainty than a single- point value for each cost [3]. Likewise, for the reactive load factor L_f a range of values can also be determined. Thus by using interval mathematics, the uncertainties associated with the capacitor allocation technique could be more effectively understood if these input parameters were treated as interval numbers whose ranges contain the uncertainties in those parameters. The resulting computations, carried out entirely in interval form, would then literally carry the uncertainties associated with the data through the analysis. Likewise, the final outcome in interval form would contain all possible solutions due to the variations in input parameters.

ALGORITHM

The implementation of the proposed optimal capacitor sizing and placement technique in interval mathematics is performed in the Matlab® environment. The steps of the algorithm are summarized as follows:

1) Run the load flow program for the original uncompensated feeder to calculate the voltages and currents at each bus using the Gauss-Seidel method.

2) Assume an initial value for the single point estimate capacitor cost K_{ck} as the average cost for all available standards for the studied feeder.

3) Let the input parameters (K_p, K_e, K_{ck} and L_f) as an interval numbers with a realistic tolerance of ±5% of their single point estimates.
4) Select a bus and apply (7), (11-13) to compute the interval capacitor size, the interval current capacitor, the interval maximum saving and the corresponding interval loss reduction respectively. Repeat this step for all buses in the feeder, except the source bus.
5) Identify the candidate bus that has the highest interval cost saving (defined here as the bus with the highest upper bound in the interval cost saving) provided that the evaluated interval loss reduction and interval capacitor size are positive.
6) Once a bus is identified as a candidate bus, determine all the standard capacitor sizes lying within the interval capacitor size at this bus. In case no standard size lies within the interval, then the one nearest to the interval is selected (i.e. the closest standard size to both the lower upper bounds of the interval). These procedures are applied at any one candidate bus selected.
7) Perform the load flow calculations, for every single standard capacitor selected earlier, to ensure that no voltage violation takes place. If there is a voltage violation for one or more standard capacitor sizes, eliminate them from further consideration. If all the capacitor sizes result in voltage violation, then go to step 5 to select the next candidate bus.
8) If there is no voltage violation, set the standard capacitor size, among the series of standard sizes in this interval, that provides the highest cost saving at this bus and take the corresponding exact capacitor cost value K_{ck}.
9) Repeat steps 4-8 to get the next capacitor bus and hence the sequence of buses to be compensated until it is found that there is no significant cost saving can be achieved by further capacitor placement.

The above algorithm can be looked upon as consisting of two nested loops. The first is a global one that loops over candidate buses to determine the interval capacitor values at all buses and the corresponding interval standard sizes. While the second is local as it searches for the optimal standard capacitor size, within an interval, at a specific candidate bus.

SIMULATION RESULTS

To illustrate the numerical algorithm presented above, a test feeder, shown in Fig. 1, whose load and feeder data are listed in [7], was investigated.

The radial distribution feeder has 9 load buses and its rated substation voltage is 23kV. The estimated typical values for K_p, K_e and L_f are \$168/kW, \$0.3/kWh, and 0.5 respectively [3]. Commercially available capacitor sizes with \$/kVAr are used in the analysis.

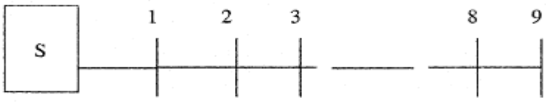

Fig. 1 Nine-bus test feeder

Applying load flow solution on this feeder, before compensation, the cost function and the total power losses are \$ 131675 and 783.8 kW respectively. The maximum and minimum bus voltage magnitudes are 0.9929 and 0.8375 p.u., where the voltage of the substation (bus number 0) is assumed to be 1 p.u., thus we have generally $0.8375 \le V_i \le 1$ p.u. The following two cases, A and B, describe the compensation procedure for the test feeder; with the input parameters K_p, K_e, K_{ck} and L_f all assumed to be interval numbers with realistic tolerances of±5%. The computations are carried using the Intlab toolbox [8].

Case A

To demonstrate the application of the proposed algorithm, equations (7), (11-13) are employed to obtain the required outcomes.
Table I shows the optimal size of a single located capacitor (Q), the maximum cost saving (ΔS) and corresponding loss reduction (ΔP), for all buses, as interval outcomes. It is noticed that the bus that provides the highest upper bound in the cost saving is bus 5 (10,320\$) which corresponds to the interval capacitor size [2309, 2838.4] KVAr. This identifies bus 5 as the first bus to be compensated. There are 3 standard sizes which fall within this range (i.e. 2400, 2550, 2700 KVAr). Computing the cost saving for each of the 3 standard sizes (provided no voltage violation occurs), it was found that the size of 2700KVAr provides the highest cost saving. The single- point estimates for the capacitor size, maximum cost saving and corresponding loss reduction at bus 5 were also computed and found to be 2560.4KVar, 8839.6\$, and 55.849kW respectively. It is clear that the estimated values of the outcomes are within the lower and upper bounds of the corresponding interval results.

Table I
Optimal sizes of singly located capacitors, cost savings and losses reduction

Bus no.	Q_c (kVAr)	DS (\$)	DP (kW)
1	[2018.1, 3057.1]	[170.16, 353.27]	[3.4883, 6.0349]
2	[2247.2, 3270.7]	[237.57, 455.33]	[4.1538, 6.937]
3	[3568.2, 4437]	[4049, 5664.6]	[27.184, 40.741]
4	[3415.8, 4215.2]	[6860.9, 9452.8]	[42.76, 63.979]
5	[2309, 2838.4]	[7548, 10320]	[45.223, 67.63]
6	[1997, 2453.4]	[7236.8, 9882.3]	[43.086, 64.431]
7	[1561.9, 1917.2]	[6729.8, 9173.7]	[39.697, 59.358]
8	[1100.6, 1349.5]	[6213.1, 8450.7]	[36.22, 54.156]
9	[836.88, 1025.5]	[5558.4, 7552.1]	[32.218, 48.171]

Additional economic benefits may be realized using the above interval outcomes of the proposed technique. It furnishes utilities with alternatives of using any available standard capacitor size, lying within the interval capacitor size outcome, together with the associated cost saving. The maximum cost saving, achieved by the selection of any of thesestandard sizes, would *certainly* have a lower limit which corresponds to the lower bound of the interval outcome. Prior knowledge of such information could be of significance in utility planning.

When the compensation procedure was continued, after placing 2700KVAr of capacitor at bus 5, it was found that interval outcomes of [410.36, 504.67] KVAr, [1250.7, 1711.6] \$, and [7.5283, 11.26] kW are obtained at bus 9. This leads to a standard set of 450KVAr at bus 9 to provide further cost saving and loss reduction. The final cost saving and loss reduction of [216.77, 327.51] \$, and [1.99, 3.04]] kW were achieved by placing a third capacitor at bus 4. The interval capacitor size at bus 4 is [618.78, 799.62] KVAr leading to a standard size of 750 KVAr. The proposed technique produces a total cost saving and a total loss reduction of [9015.52, 12358.7] $ and [54.74, 81.92] kW respectively. The estimated values of maximum cost saving and loss reduction are 10875 $ and 69.65 kW respectively by setting capacitor sizes of 2550, 450, and 900 KVAr at bus 5, 9, and 4 respectively, Again, it is noted that the single-point values are within the lower and upper bounds of the interval outcomes.

Applying load flow solution on this feeder, after compensation, the maximum and minimum bus voltage magnitudes are found to be 0.9961 and 0.8825 p.u., i.e., there is an improvement in voltage profile of about 5% in the obtained minimum voltage

Case B

In view of the fact that the algebraic structure of interval mathematics is unlike that of common single point arithmetic, interval computations may, sometimes, produce conservative bounds [6]. In order to produce better bounds (i.e. sharp bounds) of the interval outcomes the term K_p'/K_p'', appearing in the governing equations, is proposed to be of the following form:

$$\frac{K_p'}{K_p''} = 1 + \frac{K_e T\,(L_f - 1) + \dfrac{K_{ck} V_k}{2\sum_{i=1}^{k} I_{ri} R_i}}{K_p + K_e T} \qquad (15)$$

Equation (15) is then used to modify (7) and (11-13). It is expected with such modification to get sharp bounds of the interval outcomes as the appearance of the interval input parameter K_p has been reduced [6].

Table II shows the results of the modified algorithm and also its significance. For instance, at bus number 5, the earlier radius (half the interval width) of the capacitor size interval was 264.7. This value had led to the possible use of 3 standard capacitor sizes falling within that range. The corresponding radius of the interval numbers of the cost saving, and loss saving were 1385.8 and 11.2, respectively. With the modified algorithm, the radius of the interval number of the capacitor size is reduced to 8.2467. This value would lead to the use of a single standard capacitor size within that range. The corresponding radius of the interval numbers of the cost saving, and loss saving after modification become 497.96 and 0.34904, respectively.

Table II
Optimal sizes of singly located capacitors, cost savings and losses reduction using the modified technique

Bus no.	Q_c (kVAr)	DS ($)	DP (kW)
1	[2230.1, 2766.3]	[188.03, 319.67]	[4.0079, 5.3224]
2	[2483.2, 2959.6]	[262.52, 412.03]	[4.7958, 6.0912]
3	[3943.4, 4014.8]	[4474.7, 5125.6]	[33.039, 34.153]
4	[3775, 3814.1]	[7582.3, 8553.3]	[52.295, 53.332]
5	[2551.8, 2568.3]	[8341.7, 9337.6]	[55.5, 56.198]
6	[2207, 2219.9]	[7997.8, 8941.8]	[52.908, 53.512]
7	[1726.2, 1734.8]	[7437.5, 8300.7]	[48.787, 49.262]
8	[1216.4, 1221.1]	[6866.5, 7646.5]	[44.563, 44.901]
9	[924.88, 927.95]	[6142.9, 6833.4]	[39.66, 39.919]

Using the above results, a first standard capacitor size of 2550 KVAr is placed at bus 5. When the procedure is repeated, a second interval outcomes of [479.8, 483] KVAr, [1552.9, 1738.6] $, and [10.34, 10.47] kW, respectively, are achieved at bus 9. This will lead to a standard size of 450KVAr at this bus. Final cost saving and loss reduction of [352.61, 427.84] $, and [3.26, 3.53] kW are achieved by placing a third interval capacitor of [828.9, 868.7] KVAr at bus 4, leading to a standard capacitor size of 900 KVAr. The technique provides a total cost saving and total loss reduction of [10247.25, 11504.01] $ and [69.09, 70.19] kW respectively, when the above 3 standard capacitor sizes are installed. After compensation, the maximum and minimum bus voltage magnitudes are found to be 0.9961 and 0.88196 p.u. These results show that the width of the interval outcomes of the maximum cost saving and loss reduction has been reduced and their corresponding estimated values, still fall within the modified interval outcomes.

CONCLUSIONS

The capacitor sizing and placement problem is modeled using a combined heuristic and interval mathematics method. Use of interval mathematics enables the integration of the effects of parameters' uncertainties into the analysis and eliminates the need for many simulation runs. While catering for uncertainties, the method offers utilities with alternatives for selecting the standard capacitor sizes to be used and the associated costs to be saved. This enhances their ability to make informed decisions regarding installing capacitors for reactive power compensation in their distribution feeders. A procedure is devised in order to produce sharp bounds of the interval outcomes. Successful implementation of the method is described using a nine- bus test distribution feeder.

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